4.1 Polynomials

$P(x)$ is a polynomial if it can be written as a sum of products of numbers and non-negative powers of x.

$$C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \cdots + C_n x^n$$

$$(C_n \neq 0)$$

n is called the degree of this polynomial

The degree of a polynomial is the highest exponent of x.

Divide $3x^4 - 8x^2 - 11x + 1$ by $x-2$.

$3x^3 + 6x^2 + 4x - 3$ R(−5)

$x-2 \overline{)\ 3x^4 + 0x^3 - 8x^2 - 11x + 1}$

$\frac{3x^4}{x}$

$-(3x^4 - 6x^3)$

$+6x^3 - 8x^2 - 11x + 1$

$-(6x^3 - 12x^2)$

$4x^2 - 11x + 1$

$-(4x^2 - 8x)$

$-3x + 1$

$-(-3x + 6)$

-5

$(x-2)3x^3$

$3x^4 - 8x^2 - 11x + 1$ div by $x-2$ $= 3x^3 + 6x^2 + 4x - 3$ R(−5)

or

$= 3x^3 + 6x^2 + 4x - 3 + \frac{-5}{x-2}$

$3x^4 - 8x^2 - 11x + 1 = (3x^3 + 6x^2 + 4x - 3)(x-2) + (-5)$

"Division Algorithm"

$P(x) = D(x) \cdot Q(x) + R(x)$

IMPORTANT

$R(x) = 0$ means we've factored the big guy

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Divide $6x^3-4x^2+3x-2$ by $2x^2+1$

$$
\begin{array}{r}
3x-2 \\
2x^2+1 \overline{)\,6x^3-4x^2+3x-2} \\
-(6x^3\quad +3x) \\
\hline
-4x^2-2 \\
-(-4x^2-2) \\
\hline
0
\end{array}
$$

So . . .

$$6x^3-4x^2+3x-2=(3x-2)(2x^2+1)$$

Important Polynomial fact . . . .

If $(x-c)$ is a FACTOR of $P(x)$..

(c is a number)

→ $P(x)=(x-c)\,Q(x)$

→ c is a zero of $P(x)$

→ c is an x-intercept of $P(x)$

→ c is a solution of $P(x)=0$.

→ $P(c)=0$

y-intercept $=P(0)$

So . . .

$P(x) = 6x^3 - 4x^2 + 3x - 2 = (3x-2)(2x^2+1)$

$3\left(x - \frac{2}{3}\right)(2x^2+1)$

$x - c$

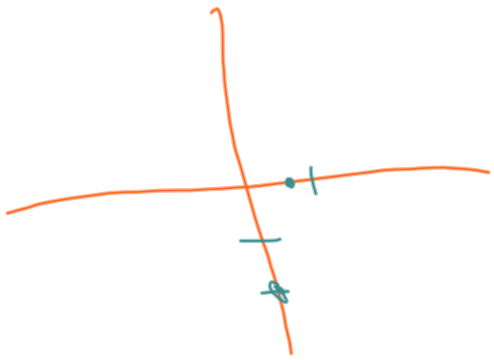
$c = \frac{2}{3}$

Give me a zero of $P(x)$ $\frac{2}{3}$

gimme a soln to $P(x) = 0$

evaluate $P\left(\frac{2}{3}\right) = 0$

Graph at least 2 pts of $6x^3 - 4x^2 + 3x - 2$

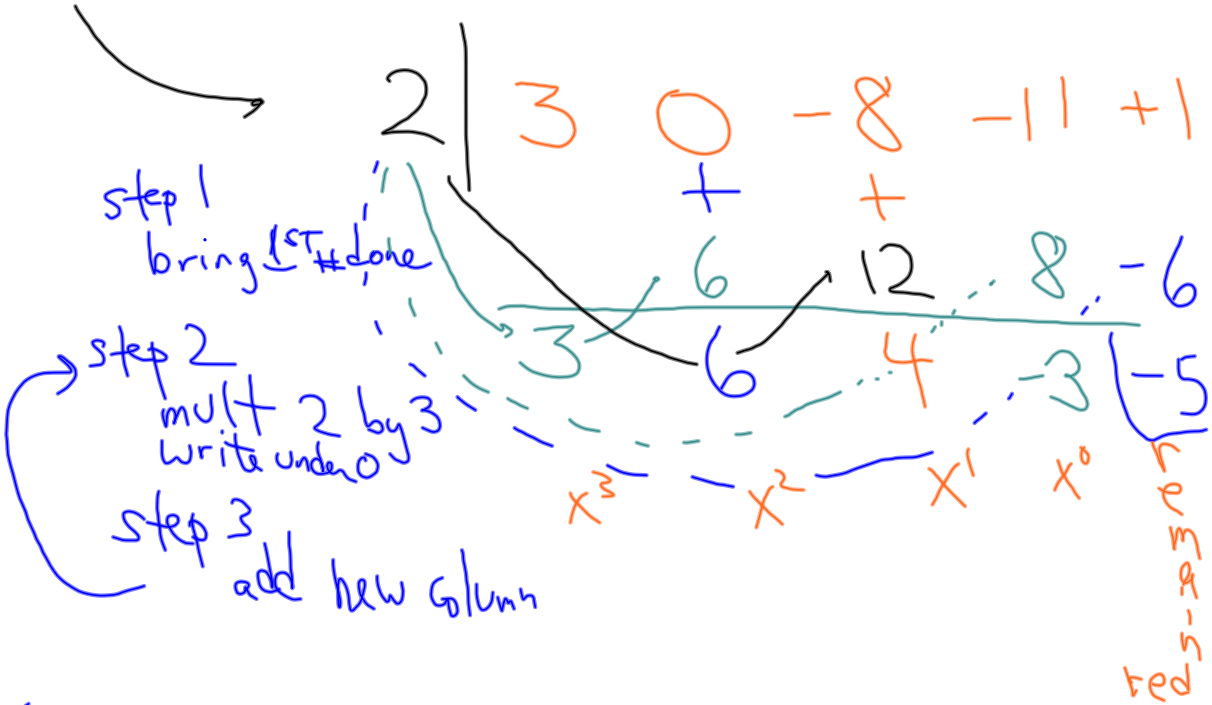


$3x^4 - 8x^2 - 11x + 1$ by $x-2$.

$(x-2)$

root $= 2$

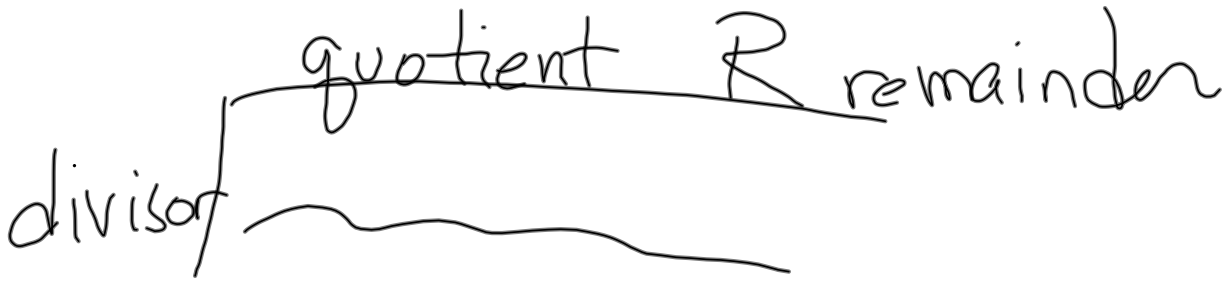
use coefficients



$(x+2)$

root $= -2$

$-2 \mid 3\ 0\ -8\ -11\ +1$


quotient R remainder
divisor